



$$\textcircled{1} \quad \pi d_1 = 2\pi r_1 \cdot \frac{\theta}{360} \quad ; \text{ the circumferences must be equal}$$

$$\textcircled{2} \quad \pi d_2 = 2\pi r_2 \cdot \frac{\theta}{360} \quad ; \text{ the circumferences must be equal}$$

$$\textcircled{3} \quad r_2 = r_1 + l \quad ; \text{ by definition}$$

$$\textcircled{4} \quad d_1 = r_1 \cdot \frac{2\theta}{360} \quad ; \text{ simplify } \textcircled{1}$$

$$\textcircled{5} \quad d_2 = r_2 \cdot \frac{2\theta}{360} \quad ; \text{ simplify } \textcircled{2}$$

$$\textcircled{6} \quad d_2 = (r_1 + l) \cdot \frac{2\theta}{360} \quad ; \text{ substitute } \textcircled{3} \text{ into } \textcircled{5}$$

$$\textcircled{7} \quad d_2 = r_1 \cdot \frac{2\theta}{360} + l \cdot \frac{2\theta}{360} \quad ; \text{ distribute } \frac{2\theta}{360}$$

$$\textcircled{8} \quad r_1 \cdot \frac{2\theta}{360} = d_2 - l \cdot \frac{2\theta}{360} \quad ; \text{ subtract } l \cdot \frac{2\theta}{360} \text{ from both sides}$$

$$\textcircled{9} \quad r_1 = d_2 \cdot \frac{360}{2\theta} - l \quad ; \text{ isolate } r_1 \text{ by dividing by } \frac{2\theta}{360}$$

$$\textcircled{10} \quad d_1 = \left(d_2 \cdot \frac{360}{2\theta} - l \right) \cdot \frac{2\theta}{360} \quad ; \text{ substitute } \textcircled{9} \text{ into } \textcircled{4}$$

$$\textcircled{11} \quad d_1 = d_2 - l \cdot \frac{2\theta}{360} \quad ; \text{ distribute } \frac{2\theta}{360}$$

(12) $l \frac{2\theta}{360} + d_1 = d_2$; add $l \frac{2\theta}{360}$ to both sides

(13) $l \frac{2\theta}{360} = d_2 - d_1$; subtract d_1 from both sides

(14) $\frac{2\theta}{360} = \frac{d_2 - d_1}{l}$; divide by l

(15) $\theta = \left(\frac{d_2 - d_1}{l} \right) \frac{360}{2}$; isolate θ

(16) $d_1 = r_1 \cdot \frac{2\theta}{360} = r_1 \cdot \frac{2}{360} \cdot \left(\frac{d_2 - d_1}{l} \right) \frac{360}{2}$; substitute (15) into (4)

(17) $d_1 = r_1 \cdot \left(\frac{d_2 - d_1}{l} \right)$; simplify

(18) $r_1 = \frac{d_1 \cdot l}{d_2 - d_1}$; isolate r_1 by dividing by $\left(\frac{d_2 - d_1}{l} \right)$

(15) and (18) give θ and r_1 in terms of known quantities d_2 , d_1 , and l